SIMULATION MODEL OF TWO-DIMENSIONAL HEAT TRANSFER IN CONTROLLED-ATMOSPHERE GRAIN BINS

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ABSTRACT

A finite-difference model was developed to simulate conductive heat transfer in both the radial and vertical directions in free-standing, controlled-atmosphere cylindrical bins of stored grain. Temperatures throughout a grain bulk can be predicted during the storage period using input data of initial grain temperatures, mean daily temperatures, solar radiation, average monthly wind velocities and thermal properties of the grain, bin structure and soil. Heat generation in the grain bulk was assumed negligible but it could be readily incorporated into the model.

To verify the model, predicted temperatures were compared with measured temperatures in 46 t of rapeseed stored for 41 months and 52 t of barley stored for 23 montns. The standard errors of estimate for the temperatures at the centres of the bins were 1.3°C for rapeseed and 1.8°C for barley. Inclusion of a submodel of natural convection currents in the grain bulk did not result in more accurate predictions of temperature and computer time was increased by a factor of about 25 over that for two-dimensional conduction alone.

Temperatures of grain in controlled-atmosphere bins predicted using both one- and two-dimensional heat transfer models were compared. The differences between the centre temperatures predicted by the two models were less than 0.2°C for diameter-to-height ratios of 0.5 or less while the differences increased to over 2.0°C for diameter-to-height ratios of greater than 1.2.

1. INTRODUCTION

Grain temperature is often a crucial factor determining the rate of deterioration of stored grain. To evaluate rapidly and inexpensively various storage methods, the temperature of the stored grain must be predicted. Muir (1970) and Yaciuk et al. (1975) presented a method of predicting temperatures in cylindrical bins having low diameter-to-height ratios. Their predictions were based on one-dimensional heat flow in a radial direction. The objective of this research project was to develop a model of two-dimensional heat flow in controlled-atmosphere cylindrical bins of grain having no forced air movement through the grain, and having high diameter-to-height ratios.

2. HEAT TRANSFER MODEL

2.1 Heat transfer in the grain bulk

The model is based on heat balance equations for heat flow in both the vertical and radial directions of a cylindrical grain bin. Temperatures throughout the bin were assumed to be symmetrical about the vertical axis and heat generation within the grain was assumed to be negligible. In developing the equations a sector of a cylindrical bin was divided into a finite number of spatial elements in the vertical and radial directions (Fig. 1).

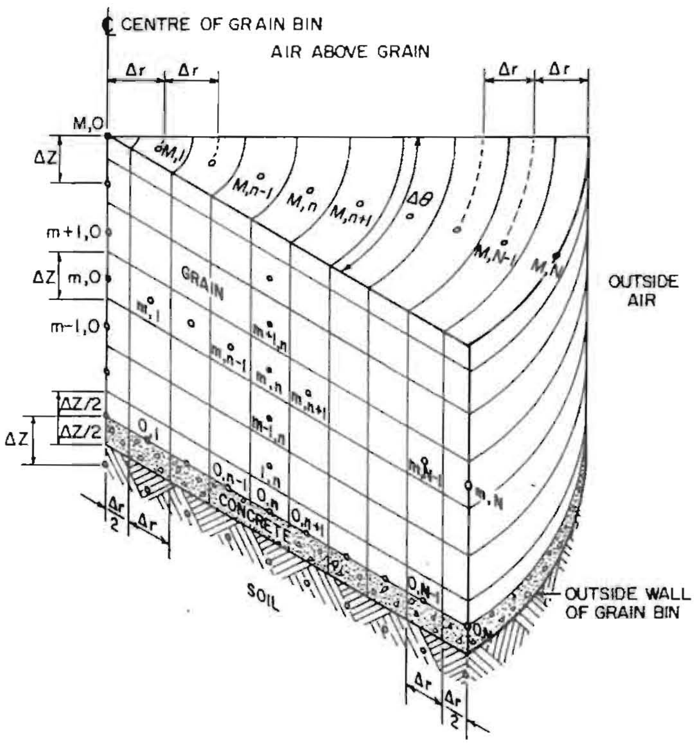

Fig. 1. Schema of a sector of a cylindrical grain bin divided into M+1 vertical and N+1 radial elements.

Following the finite difference method of Yaciuk et al. (1975), the heat balance for any interior spatial element, m,n, is (symbols are defined in Key to Symbols):

$$k_{m,n+}\left[\left(n\ \Delta r + \frac{\Delta r}{2}\right)\Delta\theta\ \Delta z\right]\left[\frac{T_{m,n+1} - T_{m,n}}{\Delta r}\right]\Delta t + k_{m,n-}\left[\left(n\ \Delta r - \frac{\Delta r}{2}\right)\Delta\theta\ \Delta z\right]\left[\frac{T_{m,n-1} - T_{m,n}}{\Delta r}\right]\Delta t$$

$$+ k_{m+,n}\left[n(\Delta r)^2\ \Delta\theta\right]\left[\frac{T_{m+1,n} - T_{m,n}}{\Delta z}\right]\Delta t + k_{m-,n}\left[n(\Delta r)^2\ \Delta\theta\right]\left[\frac{T_{m-1,n} - T_{m,n}}{\Delta z}\right]\Delta t$$

$$= n(\Delta r)^2\ \Delta\theta\ \Delta z\ c_{m,n}\ \rho_{m,n}\left(T'_{m,n} - T_{m,n}\right) \qquad (1)$$

If dimensionless moduli $U_{m,n-}$, $U_{m+,n}$ and $U_{m-,n}$ are defined in equations similar to that for $U_{m,n+}$:

$$U_{m,n+} = \frac{c_{m,n}\ \rho_{m,n}(\Delta r)^2}{k_{m,n+}\ \Delta t} \qquad (2)$$

$$\text{and } E = \frac{(\Delta r)^2}{(\Delta z)^2} \qquad (3)$$

then the predicted temperature at the end of the time increment t + Δt is:

$$T'_{m,n} = \left[\frac{2n + 1}{2n\ U_{m,n+}}\right]T_{m,n+1} + \left[\frac{2n - 1}{2n\ U_{m,n-}}\right]T_{m,n-1} + \left[\frac{E}{U_{m+,n}}\right]T_{m+1,n} + \left[\frac{E}{U_{m-,n}}\right]T_{m-1,n}$$

$$+ \left[1 - \frac{2n + 1}{2n\ U_{m,n+}} - \frac{2n - 1}{2n\ U_{m,n-}} - \frac{E}{U_{m+,n}} - \frac{E}{U_{m-,n}}\right]T_{m,n} \qquad (4)$$

If the five spatial elements, m,n-1; m,n+1; m,n; m-1,n; and m+1,n have equal thermal properties, Eq. (4) reduces to:

$$T'_{m,n} = \left[\frac{2n + 1}{2n\ U}\right]T_{m,n+1} + \left[\frac{2n - 1}{2n\ U}\right]T_{m,n-1} + \frac{E}{U}\left[T_{m+1,n} + T_{m-1,n}\right] + \left[1 - \frac{2(E + 1)}{U}\right]T_{m,n} \qquad (5)$$

$$\text{where } U = \frac{c\rho(\Delta r)^2}{k\ \Delta t} \qquad (6)$$

The equation for the predicted temperature of a centre element; m,0; with no heat flow across the centre axis can be developed using a procedure similar to that used above and to that used by Yaciuk et al. (1975):

$$T'_{m,0} = \frac{4}{U_{m,0+}}\ T_{m,1} + \frac{E}{U_{m+,0}}\ T_{m+1,0} + \frac{E}{U_{m-,0}}\ T_{m-1,0} + \left[1 - \frac{4}{U_{m,0+}} - \frac{E}{U_{m+,0}} - \frac{E}{U_{m-,0}}\right]T_{m,0} \qquad (7)$$

For a bin with constant thermal properties:

$$T'_{m,0} = \frac{4}{U} T_{m,1} + \frac{E}{U}\left[T_{m+1,0} + T_{m-1,0}\right] + \left[1 - \frac{2(E+2)}{U}\right] T_{m,0} \tag{8}$$

2.2 Heat transfer at top surface of bulk

For a spatial element at the top of the grain, M,n, the heat balance equation is:

$$k_{M,n+}\left[\left(n\Delta r + \frac{\Delta r}{2}\right)\Delta\theta\ \frac{\Delta z}{2}\right]\left[\frac{T_{M,n+1} - T_{M,n}}{\Delta r}\right]\Delta t + k_{m,n-}\left[\left(n\Delta r - \frac{\Delta r}{2}\right)\Delta\theta\ \frac{\Delta z}{2}\right]\left[\frac{T_{M,n-1} - T_{M,n}}{\Delta r}\right]\Delta t$$

$$+ \bar{h}_b\left[n(\Delta r)^2\ \Delta\theta\right]\left[T_b - T_{M,n}\right]\Delta t + k_{M-,n}\left[n(\Delta r)^2\ \Delta\theta\right]\left[\frac{T_{M-1,n} - T_{M,n}}{\Delta z}\right]\Delta t$$

$$= \frac{n(\Delta r)^2\ \Delta\theta\ \Delta z\ c_{M,n}\ \rho_{M,n}(T'_{M,n} - T_{M,n})}{2} \tag{9}$$

With the dimensionless Biot number defined as:

$$B_T = \frac{\bar{h}_b\ \Delta r}{k} \tag{10}$$

the predicted temperature of the element, for a bin of constant thermal properties is:

$$T'_{M,n} = \left[\frac{2B_T\ \Delta r}{U\ \Delta z}\right] T_b + \left[\frac{2n+1}{2nU}\right] T_{M,n+1} + \left[\frac{2n-1}{2nU}\right] T_{M,n-1} + \frac{2E}{U} T_{M-1,n}$$

$$+ \left[1 - \frac{2(E+1)}{U} - \frac{2B_T\ \Delta r}{U\ \Delta z}\right] T_{M,n} \tag{11}$$

2.3 Heat transfer at bottom surface of bulk

For the bottom layer of grain, soil temperature must be used for $T_{-1,n}$ in Eq. (1), along with appropriate thermal properties for grain, concrete and soil. The method of calculating mean thermal properties for any particular element is given by Yaciuk et al. (1975). Soil temperatures were predicted by a model developed by Singh and Schulte (1977) from equations presented by Merva (1975). The model was modified for the soil temperatures under a bin by assuming that the soil temperature profile in the horizontal direction under a bin approximates the soil temperature profile in the vertical direction.

2.4 Heat transfer at wall surface of bulk

By solving the heat balance equation the predicted temperature of an exterior surface element at the wall is:

$$T'_{m,N} = \left[\frac{8NB_W}{(4N-1)\,U_{m,N-}}\right]T_a + \left[\frac{8N-4}{(4N-1)\,U_{m,N-}}\right]T_{m,N-1} + \frac{E}{U_{m+,N}}T_{m+1,N} + \frac{E}{U_{m-,N}}T_{m-1,N}$$

$$+ \left[\frac{8N\,\Delta t}{(4N-1)\Delta r\; c_{m,N}\,\rho_{m,N}}\right]q_r + \left[1 - \frac{8NB_W + 8N - 4}{(4N-1)U_{m,N^-}} - \frac{E}{U_{m+,N}} - \frac{E}{U_{m-,N}}\right]T_{m,N} \tag{12}$$

where the dimensionless Biot number for the exterior wall surface is:

$$B_W = \frac{\bar{h}_c\,\Delta r}{k_{m,N-}} \tag{13}$$

The convective heat transfer coefficient, $\bar{h}_c$, for the exterior wall surface was calculated by the method presented by Yaciuk et al. (1975).

For a bin with constant thermal properties, Eq. (12) becomes:

$$T'_{m,N} = \left[\frac{8NB_W}{(4N-1)\,U}\right]T_a + \left[\frac{8N-4}{(4N-1)\,U}\right]T_{m,N-1} + \frac{E}{U}\left[T_{m+1,N} + T_{m-1,N}\right].$$

$$+ \left[\frac{8N\,\Delta t}{(4N-1)\Delta r\; c\rho}\right]q_r \left[1 - \frac{8NB_W + 8N - 4}{(4N-2)\,U} - \frac{2E}{U}\right]T_{m,N} \tag{14}$$

The equations for the temperatures of the four remaining elements 0,0; M,0; M,N; and 0,N are developed according to the procedures presented above.

2.5 Radiant heat transfer at wall surface of bulk

In Eqs. (12) and (14), the net radiant heat flow to the wall surface, q_r, is calculated by:

$$q_r = q_e + q_s + q_f + q_d - q_o \tag{15}$$

where:

$$q_e = \sigma\,\alpha\,F_{be}\,T_a^4 \tag{16}$$

$$q_s = \sigma\,\alpha\,F_{bs}\,T_s^4 \tag{17}$$

$$q_o = \sigma\,\alpha\,T_{m,N}^4 \tag{18}$$

and the solar radiation components q_f and q_d were calculated for Winnipeg according to the procedure explained in the next paragraph. The long-wave absorptivity and emissivity of the bin wall surface were set equal. The two shape factors, F_{be} and F_{bs}, were each set equal to 0.5. The effective sky temperature, T_s, was estimated to be 210 K (Kreith 1973).

The cylindrical bin wall was divided into 20 vertical strips with an angular width of 18°. For each hour of daylight during the period July 1957 to

December 1975 for which measured values of radiation on a horizontal surface, H, at Winnipeg were available the solar radiation on each of these vertical bin wall strips, H_v, was calculated using the equation of Jones and Kemp (1977):

$$H_v = \frac{1}{4}\left(2R_bH + 2R_bH_b + H_d + \gamma H\right) \tag{19}$$

The beam, H_b, and diffuse, H_d, components of the measured radiation were estimated using the relationships given for Winnipeg by Ruth and Chant (1976). The hourly radiation values were summed for each day and then fitted to the equation:

$$H_{vs} = a_1H_o + \frac{a_2H_s}{H_o} + a_3 \tag{20}$$

The extraterrestrial radiation, H_o, was calculated by the method given by Duffie and Beckman (1974). For the average total radiation striking all sides of a cylindrical bin at Winnipeg the coefficients for the equation are: $a_1 = 0.1152$, $a_2 = 15960$, and $a_3 = -3133$ ($R = 0.97$). This daily value of H_{vs} and the short-wave (solar) absorptivity of the bin wall material, were used to calculate the instantaneous rate of direct and diffuse radiation on the bin wall element ($q_f + q_d$) in Eq. (15). (For the average total radiation falling on the southern 55% of a bin the coefficients are: $a_1 = 0.0472$, $a_2 = 23170$, and $a_3 = -2472$ ($R = 0.97$).

The above calculation of solar radiation is directly applicable to Winnipeg only. A similar calculation procedure could be used for locations where the radiation on a horizontal surface has been measured and the relationship between diffuse and beam radiation has been developed. For locations not having measured radiation data the following equation given by Duffie and Beckman (1974) can be used:

$$H_{av} = H_o\left(b_1 + b_2\frac{s}{S}\right) \tag{21}$$

Duffie and Beckman (1974) give values for the coefficients, b_1 and b_2 for a number of locations and climate types around the world. For locations similar to Winnipeg the coefficients (Table I) based on measured sunshine hours and radiation at Winnipeg for 1972 to 1975 could be used.

2.6 Coefficients used in model

To obtain stable solutions with the finite difference model the space and time increments were chosen so that the coefficients of Eqs. (5), (8), (11), and (14) were greater than zero. Thirty-six temperature nodes, six vertically and six horizontally, were used to simulate the temperatures in a bin. A time increment of 6 h was used. The temperature at the end of each finite time increment

TABLE I

Coefficients based on measured data at Winnipeg, Canada for 1972 to 1975 that can be used to predict solar radiation on a horizontal surface

Month	b_1	b_2	R
January	0.367	0.431	0.92
February	0.384	0.451	0.92
March	0.334	0.552	0.94
April	0.250	0.578	0.94
May	0.200	0.596	0.97
June	0.198	0.575	0.96
July	0.213	0.544	0.96
August	0.201	0.548	0.95
September	0.215	0.595	0.96
October	0.231	0.522	0.95
November	0.287	0.477	0.91
December	0.350	0.419	0.88

was calculated for every spatial increment using the temperatures calculated for the end of the previous time increment.

For ambient air temperatures the daily averages of the measured maximum and minimum temperatures at the Winnipeg International Airport were used. The thermal properties of rapeseed were assumed to be: specific heat, 1700 J $kg^{-1}K^{-1}$; thermal conductivity, 0.12 W $m^{-1}K^{-1}$; and bulk density, 700 kg m^{-3} (Moysey et al. 1977, Timbers 1975). The thermal properties of barley were assumed to be: specific heat, 1560 J $kg^{-1}K^{-1}$; thermal conductivity, 0.15 W $m^{-1}K^{-1}$; and bulk density, 670 kg m^{-3} (Disney 1954).

3. MODEL OF CONVECTION CURRENTS IN GRAIN BULK

During periods of temperature variation throughout a grain bulk, convective air currents can develop carrying heat and moisture through the grain (Muir 1973). An attempt was made to model these convective currents as a refinement of the temperature prediction model. The driving force causing air movement was assumed to be the differences in force of gravity on equal volumes of air of different densities. The air density in each spatial element was calculated according to its predicted temperature. For simplification, the air was assumed to move vertically within the columns of spatial elements (Fig. 1) and to move horizontally only in the bottom row of spatial elements. The total pressure of the air at the bottom of each column was calculated by summing the forces of gravity on air masses in each element of the column. The net pressure acting on the bottom of any column was assumed to be equal to the difference between the pressure of the column itself and the average pressure of all other columns. The velocity of the air entering or leaving the bottom of the column was found from the relationship between pressure and velocity for the grain (ASAE 1979). The aeration model for near-equilibrium conditions (Thompson 1972) was used to simulate heat and moisture transfer between the grain and circulating air. The air entering each element was assumed to be in equilibrium with the upstream element. Air entering the top of a column was assumed to be 5°C above the outside ambient temperature and to have a constant relative humidity of 60%.

4. EXPERIMENTAL BINS

To develop and validate the model no temperature data were readily available for controlled-atmosphere bins. Therefore, temperatures measured by Sinha and Wallace (1977) in two cylindrical galvanized-steel bins with 5.56-m diameters located 30 km south of Winnipeg, Canada were used. Although these bins were not of the controlled-atmosphere type the joints in the wall sheets and between the wall and concrete floor were sealed against moisture movement. The presence of high CO_2 levels in the rapeseed bin (Sinha and Wallace 1977) indicates that air movement through the walls was small. It was assumed that the natural air movement through the bin structure was small enough that it had negligible effects on the grain temperatures. As is the case in controlled-atmosphere bins of dry grain it was assumed that the low level of biological activity in the bins generated a negligible amount of heat.

One bin contained 45.7 t of rapeseed to a depth of 2.7 m. The other bin contained 51.9 t of barley to a depth of 3.2 m. Temperatures were measured at the bin centre and at eight locations equidistant around three concentric circles at each of four depths in the rapeseed bin and five depths in the barley bin. Because the simulation model assumes no heat flow in a circumferential direction the eight temperatures on a concentric circle were averaged for each depth and radius combination. Because the wall temperatures were measured several centimetres from the wall these were not used. For each measurement time there were therefore 12 values for rapeseed and 15 values for barley. For the bin of rapeseed, temperature measurements for the period 13 August 1974 to 31 January 1978 (41 months) were used. For the bin of barley, temperature records for 13 August 1974 to 12 July 1976 (23 months) were used.

5. RESULTS AND DISCUSSION

5.1 Development of model

Several simulations were run for the rapeseed bin using different thermal properties. The best agreement between simulated and measured temperatures was obtained with a convective heat transfer coefficient at the top grain surface of 1.0 W $m^{-2}K^{-1}$, a temperature for the air above the grain surface of 5°C above outside ambient temperature, and 0.28 and 0.89 for long-wave and short-wave emissivities of the galvanized steel wall. These emissivity values are the ones given by Kreith (1973) for dirty galvanized iron. The model developed using the rapeseed bin data was then verified by predicting temperatures in the barley bin using the thermal properties of barley instead of rapeseed.

5.2 Predicted temperatures compared with measured temperatures

In comparing simulated and measured temperatures (Fig. 2) the standard error of estimate for the two-dimensional model was 2.0°C overall for both rapeseed and barley bins. The standard errors of estimate at the centre of the bin were 1.3°C for rapeseed and 1.8°C for barley. The maximum differences between simulated and measured results at any time during the 41 months of storage were 9.3°C at the surface, and 2.6°C at the centre of the rapeseed bin. The maximum differences were 7.0°C at 20 cm below the surface and 4.0°C at the centre of the barley bin. Increasing the number of space increments and decreasing the time interval would increase the accuracy but at the expense of increased computer time.

5.3 Convection model

Inclusion of a model of the convection currents in a grain bulk did not result in more accurate predictions of temperature. Apparently, the original assumption that convective heat transfer in the grain bulk is negligible is valid. The amount of moisture migration predicted by the model was greater than that measured and computer time was increased by a factor of about 25 over two-dimensional conduction alone.

5.4 Comparison of one- and two-dimensional models

The temperatures for the centre of grain bins predicted by one- and two-dimensional models were compared over a period of 14 to 16 mo. Grain depths were varied from 1 to 8 m in a 4-m diameter bin and 6 to 24 m in a 12-m diameter bin. For the one-dimensional model 11 temperature nodes were used. For the two-dimensional model 6 vertical and 6 horizontal temperature nodes were used. For both models the time increment was 1 h.

The differences between the centre temperatures predicted by the two models were negligible ($\leq$ 0.2°C) for diameter-to-height ratios of 0.5 or less and larger ($\geq$ 2.0°C) for ratios greater than 1.2 (Fig. 3). The primary advantages of the two-dimensional heat transfer simulation model over the one-dimensional model are its increased accuracy and increased number of locations at which temperatures are predicted. However, computer time for the two-dimensional model is increased in approximate proportion to the increase in number of temperature nodes. For most simulations the accuracy of the two-dimensional model is probably only required for bins having a diameter-to-height ratio of greater than 1.2. The two-dimensional model is also required for bins where a vertical temperature gradient is caused by heat generation or forced air movement through the grain bulk.

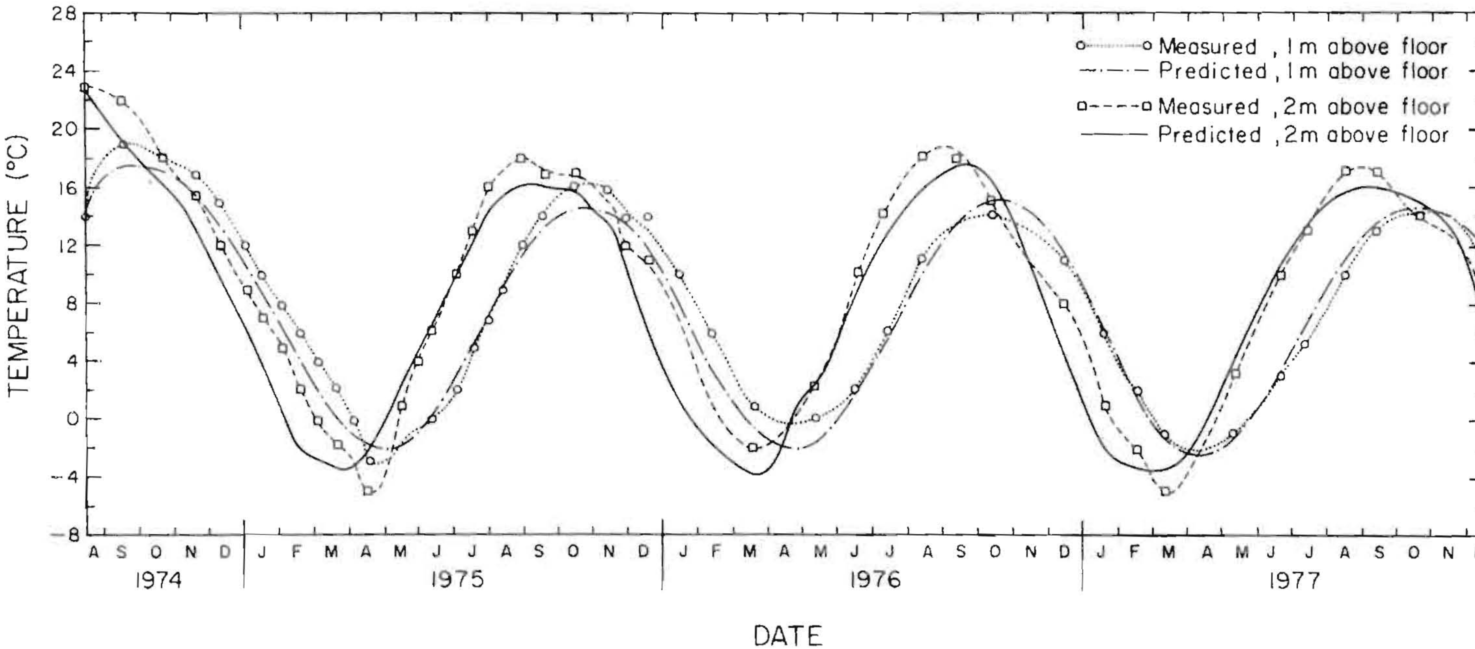

Fig. 2. Predicted and measured temperatures along the centre axis of a 5.56-m diameter cylindrical bin containing rapeseed of 2.7-m depth located 30 km south of Winnipeg, Canada.

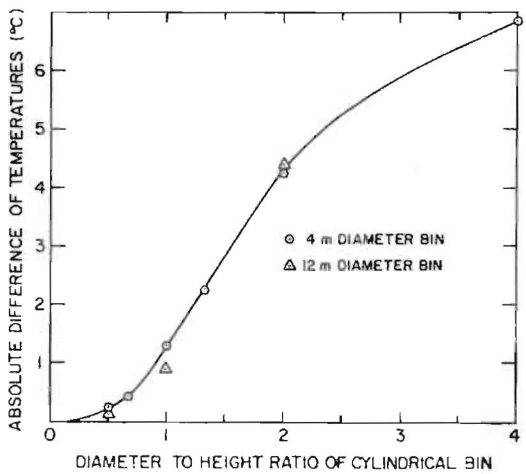

Fig. 3. Average absolute differences between temperatures at the centre of cylindrical bins predicted using one- and two-dimensional heat transfer models.

5.5 Ecological significance of bin temperature simulation

Relatively accurate prediction and confirmation of rapeseed and barley temperatures at different depths of commercial bins is of considerable ecological significance. In temperate climates, regulation of the numbers of insect and mite pests of stored grain is primarily dependent on temperature and secondarily on moisture content of the stored grain and oilseeds; other variables, such as food quality, species genotype and dockage also affect their multiplication (Sinha and Wallace 1973). Regulation of fungal populations, however, depends primarily on moisture content and secondarily on temperature (Wallace et al. 1976). Cereals and oilseeds stored in bulk on western Canadian farms retain their moisture content within a 1-3% range for several years. Distribution of the moisture content within this range, however, is uneven. Temperature fluctuations within the same bin occur over a larger range, sometimes as large as 40°C. Species of stored grain fauna and microflora exploit microenvironments with different combinations of temperature and moisture. Prior knowledge of such areas favorable for rapid multiplication of pest organisms should be an integral part of stored grain and oilseed protection strategy. For example, if bins have pockets with temperatures (over 30°C) favorable for development of the rusty grain beetle, Cryptolestes ferrugineus (Steph.), turning (Muir et al. 1977) or ventilating such grain in cold weather while the insect population is still low could prevent serious insect infestations.

KEY TO SYMBOLS

a_1, a_2, a_3	coefficients
b_1, b_2	coefficients in Eq. (18) for a given location
B_T	Biot number for top grain surface

B_W	Biot number for exterior wall surface
$C_{m,n}$	specific heat of element m,n; J $kg^{-1}K^{-1}$
F_{be}	radiation shape factor for bin-to-earth
F_{bs}	radiation shape factor for bin-to-sky
$\bar{h}_b$	convective heat transfer coefficient at top surface of bulk, $Wm^{-2}K^{-1}$
$\bar{h}_c$	convective heat transfer coefficient at exterior wall surface, $Wm^{-2}K^{-1}$
H	measured total radiation on a horizontal surface ($H_b + H_d$), W/m^2
H_{av}	average radiation on a horizontal surface for a given period of time, J/m^2
H_b	beam radiation on a horizontal surface, W/m^2
H_d	diffuse radiation on a horizontal surface, W/m^2
H_o	extraterrestrial radiation for the given location and time period, J/m^2
H_s	daily total radiation on a horizontal surface, J/m^2
H_v	radiation on a vertical surface, W/m^2
H_{vs}	daily radiation on a vertical surface, J/m^2
$k_{m,n+}$	mean thermal conductivity between nodal points m,n and n,n+1; $[Wm^{-1}K^{-1}]$
$k_{m,n-}$	mean thermal conductivity between m,n and m,n-1; $[Wm^{-1}K^{-1}]$
$k_{m+,n}$	mean thermal conductivity between m,n and m+1,n; $[Wm^{-1}K^{-1}]$
$k_{m-,n}$	mean thermal conductivity between m,n and m-1,n: $[Wm^{-1}K^{-1}]$
m	number of spatial element in vertical direction (Fig. 1)
M	number of spatial element at top surface of bulk (Fig. 1)
n	number of spatial element in radial direction (Fig. 1)
N	number of spatial element at wall surface (Fig. 1)
q_d	direct solar radiation, W/m^2
q_e	earth-to-bin radiation, W/m^2
q_f	diffuse solar radiation, W/m^2
q_r	net radiation, W/m^2
q_o	bin-to-surroundings radiation, W/m^2
q_s	sky-to-bin radiation, W/m^2
r	radial distance, m
R	correlation coefficient
R_b	cosine of angle of incidence divided by cosine of zenith angle (Duffie and Beckman 1974)
s	observed daily hours of bright sunshine for the given period of time, h
S	calculated maximum possible hours of bright sunshine for the given period of time, h
t	time, s
T_a	temperature of ambient air surrounding bin, K
T_b	temperature of air above the grain surface, K
$T_{m,n}$	temperature of element m,n at time t, K
$T'_{m,n}$	temperature of element m,n at time t + Δt, K
T_s	effective temperature of the sky, K

U	dimensionless modulus
z	vertical distance, m
α	long-wave absorptivity of bin wall material
γ	ground reflectance (0.7 for snow cover, 0.2 for no snow)
Δ	finite increment
ε	long-wave emissivity of bin wall material
θ	included angle of bin sector, rad
$\rho_{m,n}$	density of element m,n, kg m^{-3}
σ	Stefan-Boltzmann constant, 5.67×10^{-8} W $m^{-2}K^{-4}$

ACKNOWLEDGEMENTS

We thank the Natural Sciences and Engineering Research Council of Canada and the Canada Department of Agriculture for their financial assistance.

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